

***From Recurrent Neural Networks to Human Neuronal Networks and Back:
A Computational Bio-Cybernetics ****

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Introductory Remarks

An unprecedented rapid development of software for various technological applications is taking place, largely based on cognition science and pattern recognition, during the last few years. And even more so many claims have been put forward by various software applications all designated as being Artificial Intelligence (AI) developments, quite many based on Science of Neural Networks. Though, since the artificial neuron of McCullock and Pitts (1943) via Hopfield's neurons (1984) to Hagen and co-authors (Oklahoma State University, 2012) neural designs and to Kasabov spiking-neurons 'neucube' (2014) and evolving connectionist systems (2003), Science of Neural Networks has undergone developments beyond any predictions that have been put forward in due course. Rightly so Hagen and Co-authors begin their book with this very first sentence: "As your read these words you are using a complex biological neural network. You have a highly interconnected set of some 10^{11} neurons to facilitate reading, breathing, motion and thinking. Each of your biological neurons, a rich assembly of tissue and chemistry, has the complexity, if not the speed of microprocessor. Some of your neural structure was with you at birth. Other parts have been established by experience." – the quotation ends. There is no step further indicated into the other side of the complexity of neuronal and neural networks spectrum. Yet, on the other side of this spectrum Science of Neurophysiology (Gayton and Hall; 2006) yielded insights [2], [12], [39] converging closer and closer to Science of Systems Biology [9] based approach to brain's living complex neuronal network. Those appear to be rather involved concepts and ideas on the crossroad with Kolmogorov's representation superposition and Hilbert's Thirteen Problem, which appeared to yield emergence of certain delicate subtle issues (Sprecher; 2017). This paper gives one perception of these issues and a revised insight into the foundations of past developments, possibly by re-thinking the realm of recurrent artificial neural networks which possess time-varying delays within the setting of recent new stability results (Yan et al., 2015; 2016) to which the authors have been involved. Furthermore, it attempts to combine those two spectrum-ends in a cybernetic convolution thus giving new prospect for innovative findings with novel AI potential for applications.

A Background Motivation: Complex Biological versus Artificial Brains

On the other hand, recently deceased genius Stephen W. Hawking, in his 2014 arXive article on conservation of information and estimation of time for black holes [19], argued that we are facing the century of complexity in the scientific studies and its proper mathematical capturing that is soundly grounded on physics [1], [15], [16], [19-25], [33]. Before his passing Hawking also warned on considerable danger from abuse of AI-technologies, which is imminent if their underlying human background and drive are neglected. Earlier this year, in a wider societal prospect, the AI danger was pointed out in [25], [32], [34] as well. But the main issue here seems to be the interference, interaction, and interplay of energy, matter and information within the complex

networks and systems such as the human brain is [2], [12], [37] in the first place. Thus the complexity of human brain's network systemic integrity structure and integrated functioning is getting an open question of the knowledge quest, which seems a never ending story. Moreover, Hawking's warning emphasized how far reaching were the discoveries by David O. Hebb (1949) on learning organization of behaviour [17] thus also John J. Hopfield's dynamic artificial neural networks (1982, 1984), physically on sound foundations with rigorous mathematical proofs based on Lyapunov stability theory, the only universal stability theory.

The heart of each of these discoveries was built up around the idea of recurrent network structure of neurons within the functionality of which human cortex retains centrality role hence human mind too. The computational functionality of recurrent neural networks, nonetheless, implies the operating steady-state equilibrium is reached fast and first [2], [7], [45-47]. Recently, in [11] Forinto, Zalesky and Bullmore (2016) have proposed a rather innovative treatise of the fundamentals of brain network analysis approach. It appeared, the main issues evolve among the interference, interaction, and interplay of energy, matter and information within the complex networks such as the human brain [11] appears to be. It has been generally adopted to view the healthy human neuronal system for each of its life-physiology functions as a specific three-stage dynamic system with a certain internal, but outer acting, feedback with capacity of receiving stimuli (external and internal ones) and responding by relevant reaction as appropriate [10]. In turn, it seem more than natural that further progress can be achieved by observing in parallel the cybernetic phenomena of physiology functioning living neurons and computational cybernetics phenomena in complex networks systems. Functionally, the beating heart of each of these model systems is built by a recurrent network structure of neurons nonetheless.

On Controlled Creation Complex Biological Brains and Their Applications [43]

The above discussed observations, along with recent findings [35], [40-42] on controlled creation of robot cultured-neuron controllers (Warwick, 2016; Warwick et al., 2010, 2011) and on capturing the symbiotic functionality of neuron-glia cells (Dimirovski et al., 2017), a symbiotic co-existence of neural and glia cells within the same cell-compound [9], has given considerably incentives to re-visit both certain fundamental findings in neurophysiology in conjunction with Hopfield and other dynamic neural networks. It is in this way, albeit making use of the existing knowledge, it is believed this paper given new cybernetic insights into both the neuron level and the recurrent neural network level [8].

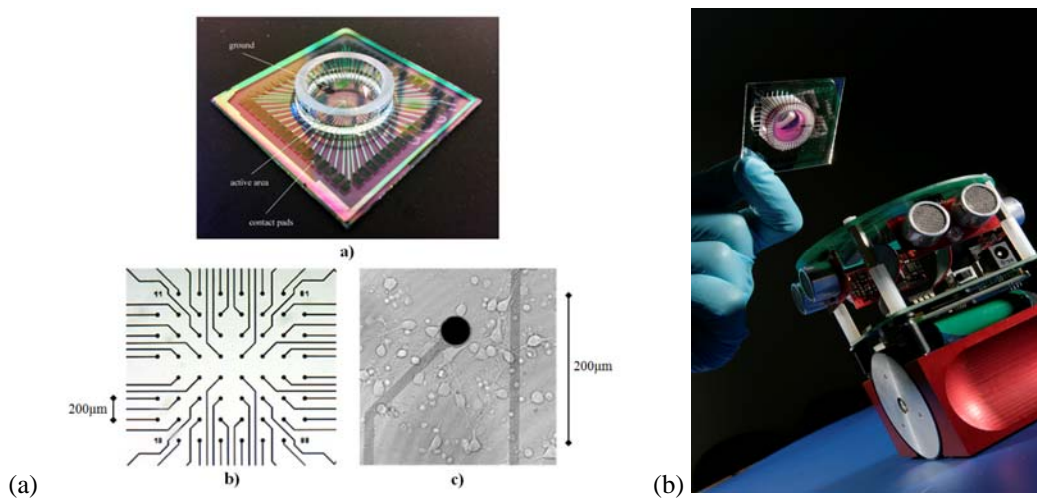


Figure 1 In Fig. 1(a) a MEA unit is depicted, showing the 30 µm electrodes which lead to the electrode column–row arrangement, where a) depicts its physics, b) shows electrode arrays in the centre of the MEA seen under an optical microscope, while c) shows MEA at x40 magnification (also showing neuronal cells in close proximity to an electrode with visible extensions and inter-connections). In the Fig. 1 (b), there is depicted the whole mobile robotic system [43].

The multi-electrode array enables voltage fluctuations in the culture (relative to a reference ground electrode outside the network) to be recorded in real-time at 59 sites out of 64 in an '8x8' array (Figure 1 a). This allows for the detection of neuronal action potentials within a 100 micro-m radius (or more) around an individual electrode. By using spike sorting algorithms [12], it is then possible to separate the firings of multiple individual neurons, or small groups of neurons, as monitored on a single electrode. As a result, multi-electrode recordings across the culture permit a picture of the global activity of the entire neuronal network to be formed. It is possible to electrically stimulate via any of the electrodes to induce focussed neural activity. The multi-electrode array therefore forms a functional and non-destructive bi-directional interface to the cultured neurons. Electrically-evoked responses and spontaneous activity in the culture (neuronal network) are coupled to the robot architecture, and thence on to the

physical robot, via a machine learning interface, which maps the features of interest to specific actuator commands. It is important to realise that the overall system employed in this experiment has been designed based on a closed-loop, modular architecture. As neuronal networks exhibit spatiotemporal patterns with millisecond precision, processing of these signals necessitates a very rapid response from neural-physiological recording and robot control systems. In recent years the study of neuronal cultures has been greatly facilitated by commercially available planar MEA systems. These consist of a glass specimen chamber lined with an 8x8 array of electrodes as shown in Fig. 1 a) It is just such one of these MEAs that we have employed in cultured-neuronal controller of a mobile robot system.. A standard MEA (Figure 1a) measures 49 mm x 49 mm x 1 mm and its electrodes provide a bidirectional link between the culture and the rest of the system. The associated data acquisition hardware includes a head-stage (MEA connecting interface), 60 channel amplifier (1200x gain; 10-3200Hz band-pass filter), stimulus generator and PC data acquisition card.

Nowadays, it is well understood that among 100 billion neurons and 1000 trillion synapse connections of human cortex's natural neural network (NNN), its essentially neuronal network possessing nonlinear dynamics may well have also billions of neurons possessing local feedbacks and featuring delays. These facts shed new light on the important relevance of recurrent dynamic networks as the fundamental cybernetic category where fundamental computational mathematical categories (Sprecher, 2016) meet the fundamental systemic categories (Dimirovski et al., 1977) within the feasible operating steady-state equilibrium, at which the proper functioning of artificial neural networks (ANN) is guaranteed. However, this implies not only recurrent neural networks but also the phenomena of time delays units with nonlinear dynamics with regard to functionally operational steady-state equilibrium (Hopfield, 1982,1984), albeit also having malfunctioning potential, which reflects the life functionalities of human cortex. These facts shed new light on the important relevance of both in recurrent neural networks (RNN) with nonlinear artificial neurons (AN) possessing time-varying delays of signal propagations within the RNN. For, these may affect considerably the functionality of operating steady-state equilibrium and cause potential malfunctioning, which appear to mimic life functionalities of human cortex. Thus, the computational functionality of all ANN including the RNN implies that they are guaranteed to first and very fast reach the operating steady-state equilibrium. These findings emphasised the importance of functional stability of such cybernetic category as recurrent neuronal-systemic structures are (Dimirovski, 2016, 2017). It has been shown into our recent articles (Yan et al., 2015; 2016) that a new insight and solutions to such a fundamental problem are feasible via employing special Lyapunov-Krasovskii functional when using Lyapunov stability theory to explore delay-dependent conditions for guaranteed fast-reaching the steady-state equilibrium. It has been proved recently the maximum bound on delays plays crucial role recurrent artificial neuronal networks provided each individual neuron retains its functional stability (Dimirovski and co-authors, 2017). This aspect has been further explored there via a kind of parallelism synergy to artificial and living neurons.

A More Comprehensive RANN with Nonlinear Neurons and Varying Time Delays [45]

Consider now the following class of models for recurrent artificial neural networks with time-varying delays

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-h(t))) + J, \quad (1)$$

along with the bound conditions observed. Other symbols denote: $z(t) = [z_1(t), \dots, z_n(t)]^T \in \mathbb{R}^n$ is a real-valued n -vector of the state variables associated with the individual neurons; $f(z) = [f_1(z_1), \dots, f_n(z_n)]^T \in \mathbb{R}^n$ is n -vector of the neuron activation functions; $J = [J_1, \dots, J_n]^T \in \mathbb{R}^n$ is the bias constant vector; $C = \text{diag}\{c_1, \dots, c_n\} \in \mathbb{R}^{n \times n}$ and A, B are the constant matrices of appropriate dimensions completing the description of this class of RANN. The delay $h(t)$ is described by a time-varying continuous function possessing the specific properties C1 and C2; see [45, 46]. Also, the activation functions $f_i(z_i(t))$, $i = 1, \dots, n$, are continuous, bounded, and satisfy the respective inequality conditions. For the stability analysis of the neural networks (1), firstly the equilibrium point z^* is shifted to origin $x = z - z^*$, $g(x) = f(x + z^*) - f(z^*)$. Then RANN model (9) can be converted into

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-h(t))) \quad (2)$$

where: $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of transformed system model $g(x(t)) = [g_1(x_1(t)), \dots, g_n(x_n(t))]^T$ with $g_j(x_j(t)) = f_j(x_j(t) + z_j^*) - f_j(z_j^*)$ and satisfying $g_j(0) = 0$ ($j = 1, \dots, n$). Functions $g_i(\cdot)$ ($i = 1, \dots, n$) now satisfy the following bound conditions:

$$k_i^- \leq \frac{g_i(u) - g_i(v)}{u - v} \leq k_i^+, \quad u, v \in \mathbb{R}, u \neq v, i = 1, \dots, n. \quad (3)$$

The following inequality

$$k_i^- \leq \frac{g_i(u)}{u} \leq k_i^+, \quad \forall u \neq 0, i = 1, \dots, n. \quad (4)$$

then holds if value $\nu = 0$ appears in (3). Thus the asymptotic stability of the operating equilibrium of considered RANN (9) coincides with the asymptotic stability of system (10).

RANN Stability Theorem. For a given positive scalar h_M , any scalars h_D^l and h_D^u with condition C1, diagonal matrices K_m , K_M , network system (1) is asymptotically stable, if there exist positive definite matrices $P \in \mathbb{R}^{5n \times 5n}$, $Q \in \mathbb{R}^{6n \times 6n}$, $R \in \mathbb{R}^{6n \times 6n}$, $N \in \mathbb{R}^{5n \times 5n}$, $Z \in \mathbb{R}^{3n \times 3n}$, $M \in \mathbb{R}^{n \times n}$, $D_i = \text{diag}\{d_{1i}, d_{2i}, \dots, d_{mi}\} \geq 0$ ($i = 1, \dots, 6$) $T_i = \text{diag}\{t_{1i}, t_{2i}, \dots, t_{mi}\} \geq 0$ ($i = 1, 2, 3$), and any matrix $W \in \mathbb{R}^{3n \times 3n}$, along with matrices $S_{ij} \in \mathbb{R}^{n \times n}$ ($i, j = 1, 2$) and matrix Π having appropriate dimensions, such that the following LMIs are computationally feasible for $h = (0, h_M)$ and for $\dot{h} = (h_D^l, h_D^u)$

$$\begin{bmatrix} (H^\perp)^T \Omega(h, \dot{h})(H^\perp) + \text{Sym}\{(H^\perp)^T \Gamma(h) \Pi^T\} & \Pi \\ * & -X \end{bmatrix} < 0, \quad (5)$$

$$X > 0, \quad \Phi > 0 \quad (6)$$

where $\Omega(h, \dot{h}) = \Sigma(\dot{h}) + h(t)\Sigma_1(\dot{h}) + (h_M - h(t))\Sigma_2(\dot{h}) + \Psi + \Theta$, $\Gamma(h) = \Pi_{10}^0 + h(t)\Pi_{10}^1 + (h_M - h(t))\Pi_{10}^2$, and the other matrices are defined in [45, 46], by means of the set of equations, with H^\perp representing the right orthogonal complement of H . Thus, original RANN has a stable operating equilibrium state.

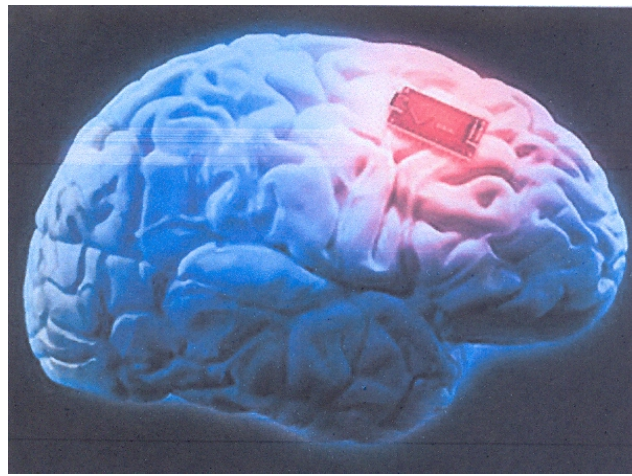


Figure 2 A Biomedical Engineering Visionary: Upgrading human intellectual capacity by employing electronic chips of brain implants

It is important to note that if and only if the comparative knowledge and understanding of both brain's natural neuronal network and the artificial neuronal network with respect to both healthy physiological as well as healthy mental functioning, then perhaps the science-fiction dream about enhancing our intellectual capacity by means of brain implants (Fig. 2; from IEEE Spectrum – The Human OS Alert, 15 November 2017) may be pursued [8], [41].

Instead of Concluding Remarks [7, 8]

The proposed talk is aimed at an elaborate presentation of those novel findings and some modification extension of existing models. In general, time-varying delays in the cortex may occur due to the cognition circumstances. If bounded time-delays take place the behavior performance appears less conservative as Lyapunov-Krasovskii delay-dependent and -independent stability criteria have proven. Thus, the maximum delay bound is an important index for understanding the underlying impact on functionality at both single-neuron local level and the level of recurrent artificial neural network (RANN) globally. Significant research efforts have been devoted to the reduction of conservatism of the delay-dependent stability criteria for the time-delay RANN. These findings emphasised the functional stability of such a bio-cybernetic category as recurrent neuronal-systemic structures capturing essential phenomena in living natural neuronal networks such as the ones in living brains. In the recent articles [45-46] there has been derived a novel insight as well as new solutions to such a fundamental problem are found (see Yan et al., 2015, 2016) via Krasovskii's extension of Lyapunov stability theory (Krasovskii, 1968) in consistence with LaSalle's invariance principle (LaSalle, 1967).

REFERENCES

- [1] F. J. Blatt, *Principles of Physics* (3rd Ed.). Boston – London Sydney – Toronto: Allyn and Beacon, 1989.
- [2] A. Brodal, *Neurological Anatomy in Relation to Clinical Medicine*. New York, NY: Oxford University Press, 1981.
- [3] D. J. Bakkum, A. Shkolnik, G. Ben-Ary, T. DeMarse, and S. Potter, “Removing some ‘A’ from AI: Embodied cultured networks. In: *Lecture Notes in Computer Science*. New York: Springer, 2004, pp. 130-145.
- [4] Q. Chang, P. Gold, “Switching memory systems during learning: Changes in patterns of brain acetylcholine release in the hippocampus and striatum in rats.” *Journal of Neuroscience*, 23, pp.3001-3005, 2003.
- [5] T. DeMarse, D., Wagenaar, A., Blau, S. and Potter, “The neurally controlled animat: Biological brains acting with simulated bodies.” *Autonomous Robots*, vol. 11, pp. 305-310, 2001.
- [6] M. Chiappalone, A. Vato, L. Berdondini, M. Koudelka-Hep, S. Martinoia, “Network dynamics and synchronous activity in cultured cortical neurons.” *International Journal of Neural Systems*, vol. 17, is. 2, pp. 87-103, 2007.
- [7] G. M. Dimirovski, R. Wang, B. Yang, “Delay and recurrent neural networks: Computational cybernetics of systems biology?” In *Proceedings IEEE SMC 2017 on Systems, Man and Cybernetics*, (A. Basu, General Chair; W. Pedricz and I. Cheng, Program Chairs), Banff, Canada, 4-8 October, Paper MB-159_FrA14.3. University of Alberta and the IEEE, Piscataway, NJ., 2017.
- [8] G. M. Dimirovski, Further Exploration into the Complexity Symbiosis of Neron-Glia Cells. *Report LNN-ANN-GMD2*. DOU, Istanbul, 2017.
- [9] G. M. Dimirovski, A Preliminary Study of the Complexity Symbiosis of Neron-Glia Cells. *Report LNN-ANN-GMD1*. DOU, Istanbul, 2016.
- [10] G. M. Dimirovski, An Overview of Fascinating Ideas on Complexity and Complex Networks and Systems in Computational Cybernetics. In *Proceedings IEEE EUROCON 2017 on Smart Technologies*, 6-8 July, Ohrid, R. Macedonia, pp. 650-664. European Union IEEE and the IEEE, Piscataway, NJ, July 2017.
- [11] A. Fornito, A. Zalesky, E. Bulmore, *Fundamentals of Brain Network Analysis*. New York, NY: Academic Press, 2016.
- [12] A. C. Guyton, J. E. Hall, *Medical Physiology* (11th Ed.). Philadelphia, PA: Elsevier Saunders, 2006, Chapters 45, 46, and 54-58.
- [13] M. T. Hagen, H. B. Demuth, M. H. Beale, Orlando De Jesus, *Neural Network Design* (Second Edition). Oklahoma State University, Stillwater, OK, 2012.
- [14] P. G. Haydon, “Glia: Listening and talking to the synapse.” *National Reviews of Neuroscience*, vol 2, pp. 844-847, 2001.
- [15] S. Haykin, *Neural Networks and Learning Machines* (3rd Ed.). Upper Saddle River, NJ: Pearson education, Inc., 2009.
- [16] R. Hecht-Nilsen, “Kolmogorov’s mapping neural network existence theorem.” In *Proceedings of the First IEEE International Conference on Neural Networks*, San Diego, CA. New York, NY: The IEEE, 1987, vol. III, pp. 11-14.
- [17] S. W. Hawking, “Conservation of information and estimation of time for black holes.” *ArXive*, arXive-paper pp. 1-3, Jan. 2014.
- [18] T. M. Heseks, B. Kappen, “Learning processes and neural networks.” *Physics Reviews*, A44, pp. 2718-2726, 1991.
- [19] J. J. Hopfield, “Neural networks and physical systems with emergent collective computational properties.” *Proceedings of the National Academy of Sciences*, vol. 79, pp. 2554-2558, 1982.
- [20] J. J. Hopfield, “Neurons with graded responses have collective computational properties like those of two-state neurons.” *Proceedings of the National Academy of Sciences*, vol. 81, pp. 3088-3092, 1984.
- [21] F. C. Hoppensteadt and E. M. Izhikievich, *Weakly Connected Neural Networks*. New York, NY: Springer-Verlag, 1997.
- [22] E. M. Izhikievich, “Which model to use for spiking neurons?” *IEEE Transactions on Neural Networks*, vol. 15, pp. 1063-1070, 2004.
- [23] Y. Jing, D. Wang, G. M. Dimirovski, “Collective adaptation evolution of weighted complex networks: On Synchronizability dependence.” In *Proceedings of the 13th IEEE International Conference on Control and Automation, ICCA 2017* (L. Xia and J. Stefanovski, General Chairs; L. Lu and M. Stankovski, Program Chairs), Ohrid, Macedonia, 3-5 July. IEEE Control Systems Chapter of Singapore, ETAI Society of Macedonia, and the IEEE, Piscataway, NJ, July 2017, , pp. 88-93.
- [24] N. N. Kasabov, “Neucube: A spiking neuronal network architecture for mapping, learning and understanding spatio-temporal brain data.” *Neural Networks*, vol. 52, pp. 62-76, 2014.
- [25] N. Kasabov, *Evolving Connectionist Systems: Methods and Applications in Bioinformatics, Brain Study and Intelligent Machines*. London – New York – Heidelberg, Springer Verlag, 2003.
- [26] T. Li, T. Wang, A. Song, S. Fei, “Combined convex technique on delay-dependent stability for delayed neural networks. *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, pp. 1459-1466, 2013.
- [27] Z. Liu, J. Yu, D. Xu, “Vector Wirtinger-type inequality and the stability analysis of delayed neural network.” *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, pp. 1247-1257, 2013.
- [28] N. N. Krasovskii, *Control Theory of Motion (in Russian; Teoriya upravleniya dvizheniyem)*. Moskva, USSR: “Nauka”, 1968.
- [29] J. P. LaSalle, An Invariance Principle in the Theory of Stability. In: J. K. Hale and J. P. LaSalle, Editors, *Differential Equations and Dynamic Systems*. New York, NY: Academic Press, 1967, pp. 277-286.
- [30] W. S. McCulloch and W. Pitts, “A logical calculus of the ideas immanent in nervous activity.” *Bulleting of Mathematical Biophysics*, vol. 5, pp. 115-133, 1943.
- [31] P. Marks, “Rat-brained robots take their first steps.” *New Scientist*, vol. 199 (2669), pp. 22-23, 2008.
- [32] V. B. Rao, *C++ Neural Networks and Fuzzy Logic*. IDG Books Worldwide, Inc.- M&T Books, 1995.
- [33] W. Rall, “Membrane potential transients and membrane time constants of motoneurons.” *Experimental Neurology*, vol. 2, pp. 503-532, 1960.
- [34] S. Russell, P. Norvig, *Artificial Intelligence – A Modern Approach* (Third edition). Prentice Hall – Pearson Education Inc. Upper Saddle River, NJ, 2010.
- [35] M. Spencer, J. Downes, D. Xydas, M. Hammond, V. Becerra, K. Warwick, B. Whalley, S. Nasuto, “Multiscale evolving complex network model of functional connectivity in neuronal cultures.” *IEEE Transactions on Biomedical Engineering*, vol. 59, no.1, pp. 30-34, 2012.
- [36] D.A.Sprecher, *From Algebra to Computational Algorithms: Kolmogorov and Hilbert’s Problem 13*. Boston, MA: Docent Press, 2017.
- [37] G. Thews, E. Mutschler, and P. Vaupe, *Human Anatomy, Physiology, and Pathophysiology*. Amsterdam, NL: Elsevier Science, 1985.
- [38] S. Talwar, S. Xu, E., Hawley, S. Weiss, K., Moxon, J. Chapin, J., “Rat navigation guided by remote control.” *Nature*, vol. 417, pp. 37-38, 2002.
- [39] G. Thews, E. Mutschler, and P. Vaupe, *Human Anatomy, Physiology, and Pathophysiology*. Amsterdam, NL: Elsevier Science, 1985.

- [40] K. Warwick, D. Xydas, S. J. Nasuto, V. M. Becerra, M. W. Hammond, J. H. Downes, S. Marshall, and B. J. Whalley, "Controlling a mobile robot with a biological brain." *Defence Science Journal*, vol. 60, no. 1, pp. 5-14, 2010.
- [41] K. Warwick, "Implications and consequences of robots with biological brains." *Ethics and Information Technology*, vol.12,3, pp.223-234, 2010.
- [42] K. Warwick, S. Nasuto, V., Becerra, and B. Whalley, Experiments with an In-vitro Robot Brain, Computing with Instinct. In Y. Cai, Editor, *Lecture Notes in Computer Science*, Vol. 5897. Berlin, DE: Springer Verlag, 2011, Ch. 1, pp. 1-15.
- [43] K. Warwick, Creating and Controlling Complex Biological Brains. In G. M. Dimirovski, Editor, *Complex Systems: Relationships between Control, Communicationns and Computing*. Cham, CH: Springer International AG, 2016, Chapter 2.1, pp. 133- 148.
- [44] D. Xydas, J. H. Downes, M. Spencer, M. W. Hammond, S. J. Nasuto, B. J. Whalley, V. M. Becerra, and K. Warwick, "Revealing ensemble state transition patterns in multi-electrode neural recordings using hidden Markov models." *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 19, no. 4, pp. 345-355, 2011.
- [45] B. Yang, R. Wang, P. Shi, G. M. Dimirovski, "New delay-dependent stability criteria for recurrent neural networks with time-varying delays." *Neurocomputing*, vol. 151, pp. 1414-1422, 2015.
- [46] B. Yang, R. Wang, G. M. Dimirovski, "New delay-dependent stability neural networks with time-varying delays via a novel partitioning method." *Neurocomputing*, vol. 173, pp. 1017-1027, 2016.
- [47] J. Zhao, G. M. Dimirovski, "Quadratic stability of switched nonlinear systems." *IEEE Transaction of Automatic Control*, vol. 49, no. 4, pp. 574-578, 2004.